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A Unified Statistical Evaluation Method for the Noise Reduction Effect of Barriers

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ABSTRACT

For a stationary random input noise of arbitrary non-Gaussian distribution attenuated by passing through a sound insulation barrier, a unified method of evaluating the output level distribution at an observation point is theoretically deduced. The validity of the theoretical deduction is experimentally confirmed using observed noise attenuation data. The experimental results are in good agreement with the theoretically predicted probability curves.

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1. INTRODUCTION

In the practical engineering field of noise control, a sound insulation barrier is very often constructed to produce attenuation of acoustic noise. The acoustical design and/or the evaluation problem of barriers have been already considered by many investigators.[1-4] Almost all of these studies, however, were confined only to the effects on deterministic signals or gross average evaluations of shielding effects.

The practical problem of the acoustical design and/or the evaluation of sound insulation barrier, needs to take the following points into consideration:

(i) In an actual living noise environment, the noise fluctuation emitted from a sound source shows very often an irregular time pattern with the intricate ups and downs, and exhibits various kinds of probability distribution forms apart from a standard Gaussian distribution form. Furthermore, the energy associated with a particular frequency band fluctuates with the lapse of time.

(ii) The statistics such as median and L_X sound level (like L_5, L_{10}, L_{50} and L_{90}) defined as the (100-x) percentage point of sound level distribution, as well as the lower order statistics like L_{eq} (substantially the 1st order statistics of energy) are very important for the actual noise evaluation and regulation problems. Thus, it is necessary to obtain an explicit expression of the output noise level distribution function.

(iii) It is important to relate the output noise evaluation indices L_X to the statistical properties (such as, the probability distribution characteristics, frequency spectrum and its temporal change, etc.) of the noise fluctuation emitted from sound sources and the frequency characteristics of the sound insulation barrier.

In this paper, a unified statistical evaluation method for predicting the noise level distribution (or noise energy distribution) is derived for the case of an arbitrary probability distribution for the random noise incident on the barrier. The emphasis in the present study is focused on how to predict the noise evaluation indicies such as L_X and/or L_{eq} using information on the statistical properties of the random noise sound source and the frequency characteristics of the sound insulation barrier. Finally, the validity of the present theory is experimentally confirmed by applying it to actually observed data. The experimental results are in good agreement with the theoretically predicted probability curves.

2. THEORETICAL CONSIDERATION

2.1 Explicit Expression of Noise Energy Distribution

To predict the noise evaluation index, L_X, it is first necessary to find an explicit expression for the noise level or noise energy distributions. Various kinds of approaches can be used to establish the form of this probability distribution. To establish a unified evaluation method, a generalized formulation for the probability expression is employed. In order to find out this universally-applicable probability expression for various kinds of random phenomena, it is necessary to employ a statistical series expansion type probability expression having many parameters. That is, the general expression for an arbitrary distribution is a series expansion probability expression taking the Gamma distribution function as the first term in the expansion. The effect of the statistical properties of the noise energy and the frequency characteristics of the sound insulation barrier on the resultant noise level (or noise energy) distribution at an observation point will be reflected in the parameters contained in the postulated probability expression. The following discussion will present the above-mentioned probability expression.

Paying special attention to the requirement that noise energy, E, fluctuates always in a non-negative region $[0, \infty)$, the probability density function, P(E), of noise energy fluctuations can be reasonably expressed in the general form of a statistical Laguerre expansion series[5]:

$$P(E) = \frac{1}{\Gamma(m) S^m} e^{-E/S} E^{m-1} \left[1 + \sum_{n=3}^{\infty} A_n L_n^{(m-1)} \left(\frac{E}{S}\right) \right] \tag{1}$$

with $\quad m = \dfrac{\langle E \rangle^2}{\langle (E - \langle E \rangle)^2 \rangle}, \quad S = \dfrac{\langle E \rangle}{m}$

(2)

and $\quad A_n \triangleq \dfrac{\Gamma(m) n!}{\Gamma(m+n)} \langle L_n^{(m-1)} \left(\dfrac{E}{S}\right) \rangle,$

Where $L_n^{(m-1)}(\cdot)$ denotes the associated Laguerre's polynomial. And also, $\langle * \rangle$ denotes an averaging operation with respect to the random variable $*$. By using the following relationship derived from the definition of the associated Laguerre's polynomial[5]:

$$P_\Gamma(Y,m) L_n^{(m-1)}(Y) = \frac{\Gamma(m+n)}{\Gamma(m) n!} \left(\frac{d}{dY}\right)^n P_\Gamma(Y, m+n) \tag{3}$$

with $\quad P_\Gamma(Y,m) \triangleq \dfrac{1}{\Gamma(m)} e^{-Y} Y^{m-1}, \qquad (4)$

(standard Gamma distribution function)

Eq. (1) can be rewritten as:

$$P(E) = P_\Gamma(E; m, S) + \sum_{n=3}^{\infty} B_n \left(\frac{d}{dE}\right)^n P_\Gamma(E; m+n, S) \tag{5}$$

with $\quad P_\Gamma(E; m, S) \triangleq \dfrac{1}{\Gamma(m) S^m} e^{-E/S} E^{m-1} \qquad (6)$

(Gamma distribution function)

and $\quad B_n \triangleq S^n \langle L_n^{(m-1)} \left(\dfrac{E}{S}\right) \rangle. \qquad (7)$

From Eq. (5), one can find that the probability density expression for a non-negative random variable of arbitrary distribution type can be always expressed in a universal form in which the well-known Gamma distribution function is the first expansion term and its successive derivatives are involved in the second and higher expansion terms. The various statistical properties of random variables are reflected in the values of each expansion coefficient.

Thus, the cumulative probability distribution function, Q(E) ($\triangleq \int_0^E P(E) dE$), of noise energy fluctuation, which is very important for the purpose of finding an arbitrary L_X sound level, can be derived as follows:

$$Q(E) = \int_0^E P_\Gamma(E;m,S)dE + \sum_{n=3}^{\infty} B_n \left(\frac{d}{dE}\right)^{n-1} P_\Gamma(E;m+n,S)$$

$$= \int_0^E P_\Gamma(E;m,S)dE + P_\Gamma(E;m+1,S)\sum_{n=3}^{\infty} C_n L_{n-1}^{(m)}\left(\frac{E}{S}\right). \tag{8}$$

with $$C_n \triangleq \frac{S\Gamma(m+1)(n-1)!}{\Gamma(m+n)} < L_n^{(m-1)}\left(\frac{E}{S}\right) > \tag{9}$$

Hereupon, the following relationship derived from Eq. (3):

$$\frac{\Gamma(m+1)(n-1)!}{S^{n-1}\,\Gamma(m+n)}\left(\frac{d}{dE}\right)^{n-1} P_\Gamma(E;m+n,S) = P_\Gamma(E;m+1,S)L_n^{(m)} \tag{10}$$

has been used.

2.2 Relationship between Frequency Characteristics of Barrier and Parameters of Energy Probability Expression

In order to determine the above parameters m, S and C_n contained in the cumulative probability distribution expression, Eq. (8), it is first necessary to calculate the pth order moment of the noise energy, E, at an observation point. Now, let E_i ($i=1, 2, \ldots, N$) be the input noise energy fluctuation existing in the ith frequency band for random noise emitted from the sound source, and let the transfer coefficient a_i ($i=1, 2, \ldots, N$) denote the energy frequency characteristic of a sound insulation barrier at the center frequency, f_{ci}, of the ith octave band (or one-third-octave band). Based on the additive property of energy quantities, the output noise energy fluctuation, E, at an observation point can be related to the energy in each frequency band by:

$$E = \sum_{i=1}^{N} a_i E_i. \tag{11}$$

Thus, the pth order moment, $<E^P>$, for E can be estimated as:

$$<E^P> = \sum_{i+j+\ldots+n=p} \frac{P!}{i!j!\ldots n!} a_1^i a_2^j \ldots a_N^n < E_1^i \cdot E_2^j \ldots E_N^n >, \tag{12}$$

by using the statistical property of noise energy fluctuation, E_i ($i=1, 2, \ldots, N$), and the frequency characteristic, a_i, of the sound insulation barrier. Thus, three parameters m, S and C_n in Eq. (8) can be directly determined by substituting Eq. (12) into Eqs. (2) and (9).

2.3 Estimation of Frequency Characteristics of Sound Insulation Barrier

Now, let us consider the sound insulation barrier shown in Fig. 1. The Fresnel number,

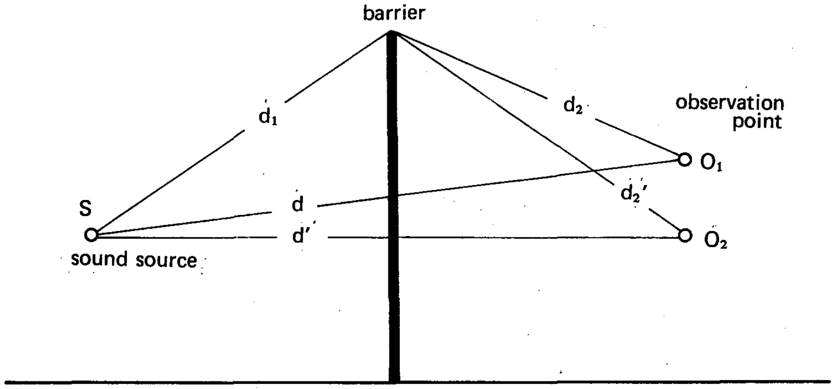

Fig. 1 Arrangement of sound source, barrier and observation points.

N_i, at a center frequency, f_{ci}, of the ith octave band (or one-third-octave band) can be determined as[1]:

$$N_i = 2\delta f_{ci}/c \ , \quad \delta = d_1 + d_2 - d, \tag{13}$$

where c is the speed of sound. As is well-known, the sound attenuation, ΔL_i, due to the construction of barrier, is predicted by use of the so-called Maekawa's acoustical evaluation chart for a barrier based on a value of N_i (e.g. see Ref. 2)). Thus, the energy frequency characteristics, a_i, of sound insulation barrier can be easily estimated as follows:

$$a_i = 1/10^{\Delta L_i/10} \ (i = 1, 2, \ldots, N). \tag{14}$$

In this case, E_i (i=1, 2, . . . , N) shown in Eq. (11) is the noise energy fluctuation existing in the ith frequency band-width at an observation point O_1 or O_2, before the sound insulation barrier is constructed. Of course, it is possible to estimate experimentally this frequency characteristic, a_i, from the actually observed data. The main purpose of the present study, however, is to predict theoretically the noise probability distribution after construction of the sound insulation barrier. The experimentally measured frequency characteristics will therefore not be used in the calculations.

3. EXPERIMENTAL CONSIDERATION

The experiment was done at night (20.00 pm ~ 03.00 am) in a playground to avoid the effect of surrounding background noise. Figure 2 shows the layout of the sound source, the

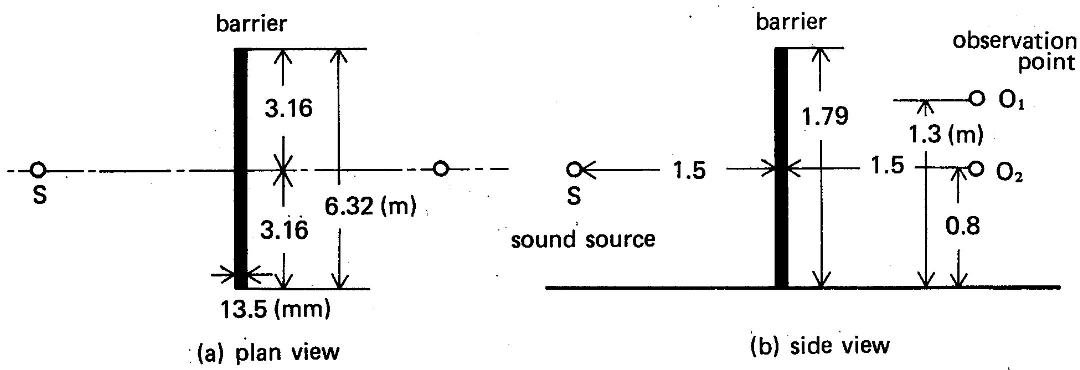

Fig. 2 Layout of sound source, barrier and observation points.

barrier and two observation points. The barrier is made of a plywood panel (height: 1.79 m, width: 6.32 m and thickness: 13.5 mm). The block diagram of the experimental arrangement is shown in Fig. 3.

Using a band-pass filter and amplifier, a road traffic noise wave (road traffic noise is used as one typical example of actual random noise fluctuation of arbitrary distribution type) recorded in advance on data recorder 1 was supplied to the loudspeaker. The received acoustic waves by

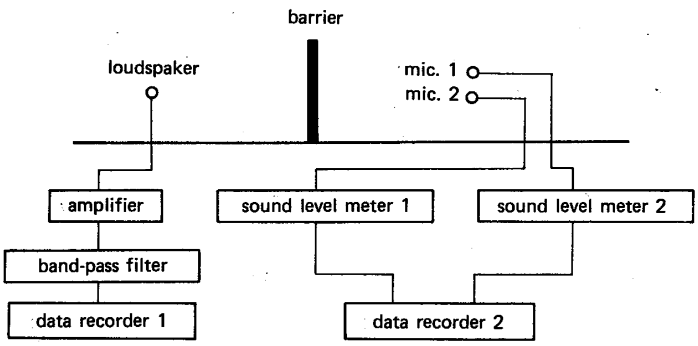

Fig. 3 Block diagram of experimental arrangement.

microphones 1 and 2 are both recorded using data recorder 2. The averaged distribution of noise energy, $<E_i>$ $(i=1, 2, \ldots, 5)$, existing in the ith frequency band of noise fluctuation radiated from the loudspaker is shown in Fig. 4. To simplify the experimental procedures, only octave-band analysis was used (accordingly, the value of N in Eq. (11) is equal to 5).

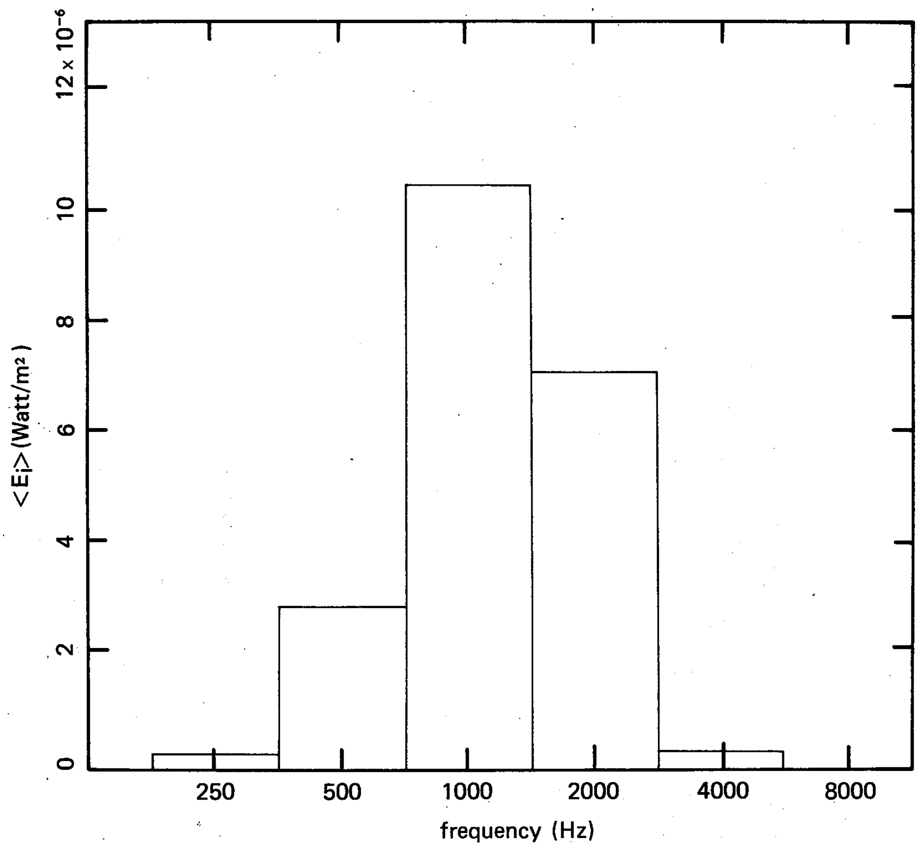

Fig. 4 The averaged distribution of frequency spectrum for energy fluctuation of road traffic noise.

The frequency characteristic, a_i, of barrier estimated by using Maekawa's evaluation chart[2] is shown in Table 1.

Figure 5 shows a comparison between the theoretically predicted curves using Eq. (8) and the experimentally sampled values for the cumulative distribution of noise energy fluctuation,

Table 1 Energy frequency characteristics a_i of barrier.

1/1 octave band center frequency (Hz)	h=0.8 (m)	h=1.3 (m)
250	0.07771	0.19498
500	0.04023	0.05370
1000	0.01943	0.04897
2000	0.00996	0.02754
4000	0.00479	0.01479

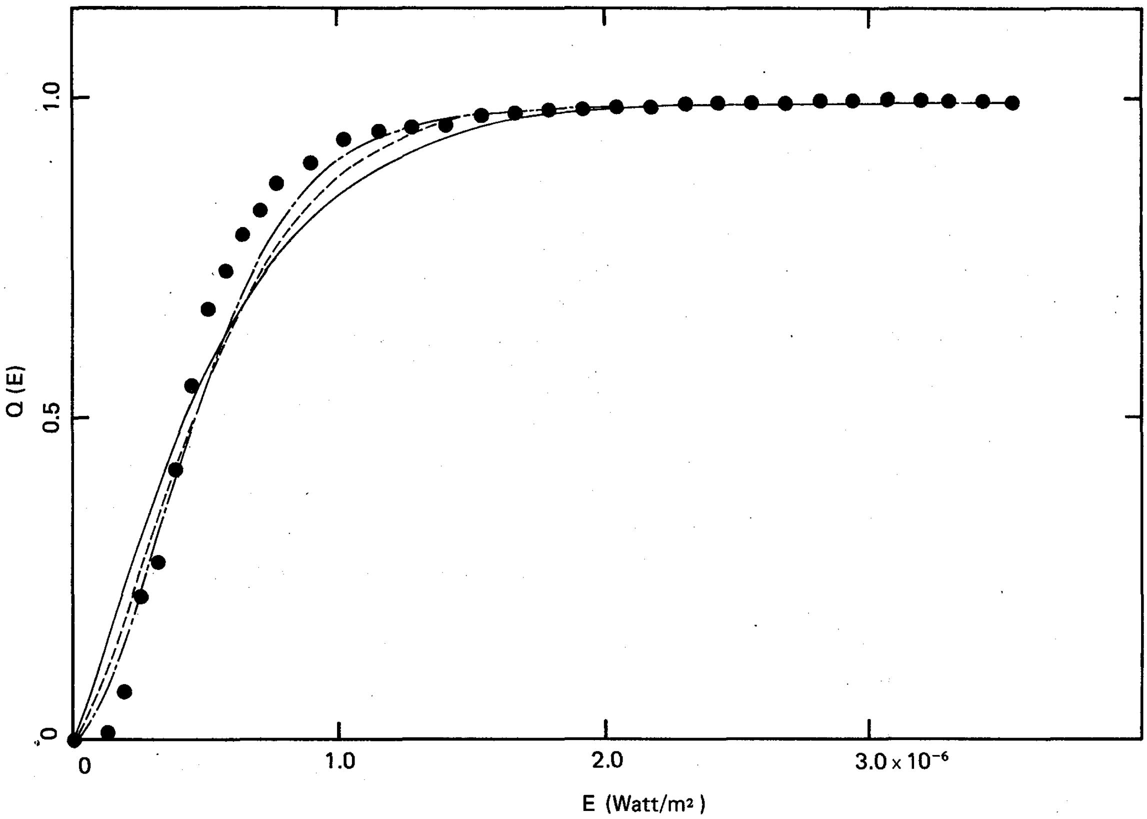

Fig. 5 Comparison between theoretically predicted curves and experimentally sampled points for cumulative noise energy distribution (height of observation point: 0.8 m). Experimentally sampled points are marked by ●, and theoretically predicted curves by use of Eq. (8) are respectively shown as ——, the first term of Eq. (8); -----, the first approximation; — · —, the second approximation.

in the case when the height of the observation point is 0.8 m (after constructing the barrier). The same experimental results are shown in Fig. 6 in the form of a noise level distribution together with the data observed before constructing the sound insulation barrier. In another case, the height of the observation point was 1.3 m, and a comparison between theory and experiment for the noise energy distribution is shown in Fig. 7. The same results for noise level distribution are shown in Fig. 8, together with the data on the input. From these figures, it is obvious that the theoretical curves are closer to the experimentally sampled values as the number of correction term increases. Furthermore, it must be noticed that the prediction error of the present method

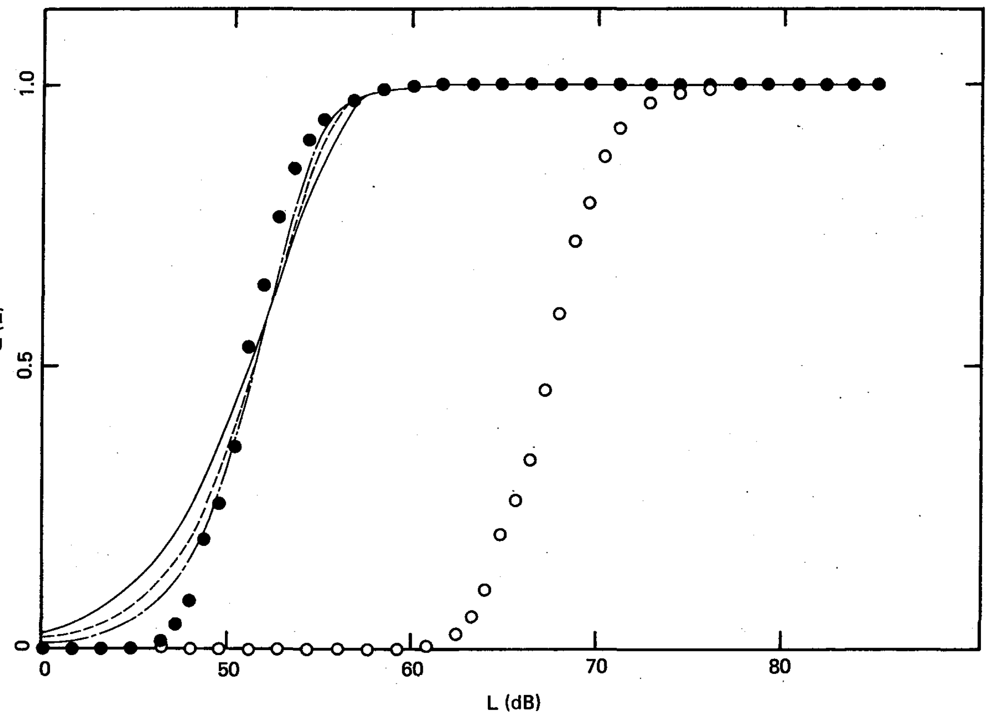

Fig. 6 Comparison between theoretically predicted curves and experimentally sampled points for cumulative noise level distribution (height of observation point : 0.8 m). Experimentally sampled points are marked by ●, and theoretically predicted curves by use of Eq. (8) are respectively shown as ——, the first term of Eq. (8) ; ------, the first approximation ; — - —, the second approximation. Experimentally sampled points observed before constructing barrier are marked by ○.

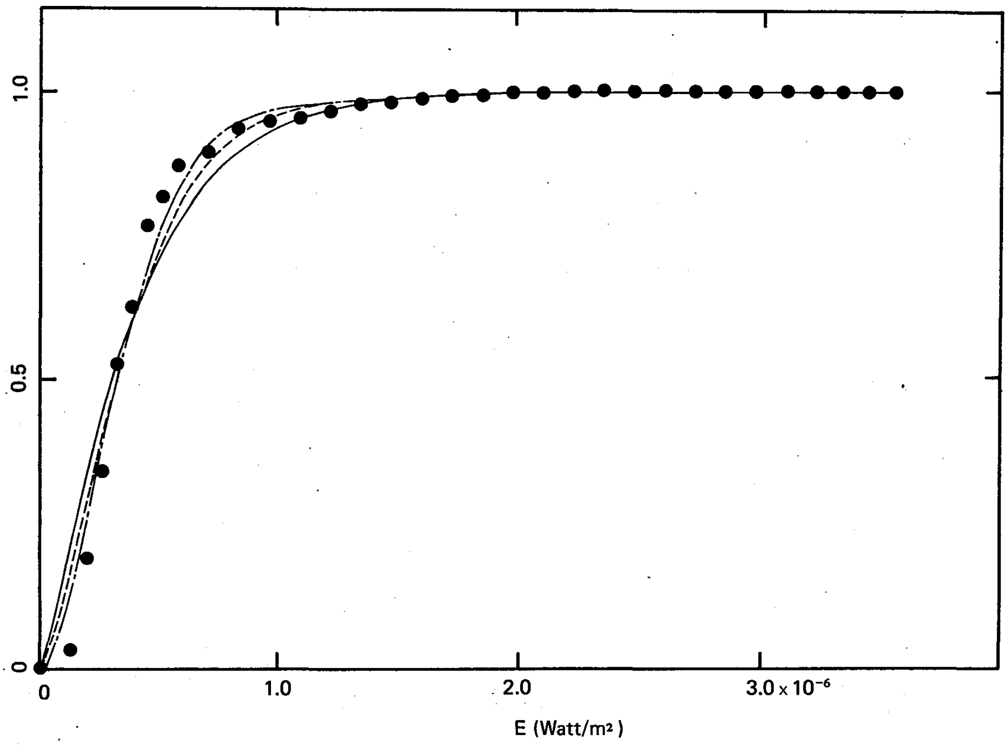

Fig. 7 Comparison between theoretically predicted curves and experimentally sampled points for cumulative noise energy distribution (height of observation point : 1.3 m). Experimentally sampled points are marked by ●, and theoretically predicted curves by use of Eq. (8) are respectively shown as ——, the first term of Eq. (8) ; ------, the first approximation ; — - —, the second approximation.

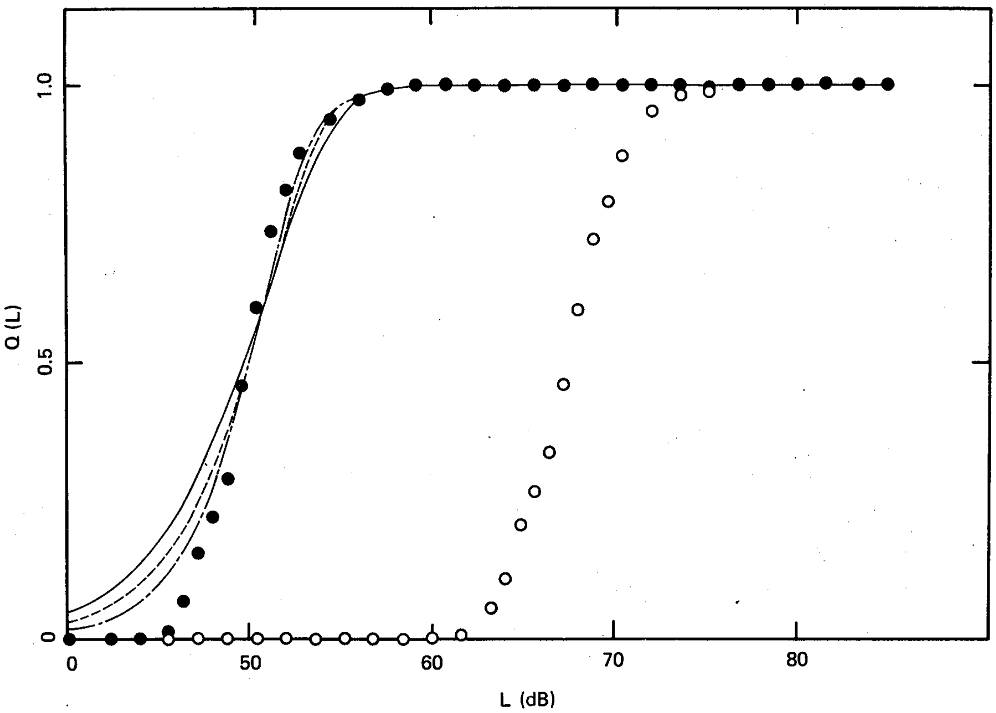

Fig. 8 Comparison between theoretically predicted curves and experimentally sampled points for cumulative noise level distribution (height of observation point: 1.3 m). Experimentally sampled points are marked by ●, and theoretically predicted curves by use of Eq. (8) are respectively shown as ——, the first term of Eq. (8); ------, the first approximation; —·—, the second approximation. Experimentally sampled points observed before constructing barrier are marked by ○.

for noise evaluation indices, L_5, L_{10} and L_{50} usually used in the noise evaluation and/or the regulation problem is about ± 1 dB.

4. CONCLUSION

A unified statistical method for predicting the probability distribution for either noise energy or noise level after constructing a sound insulation barrier has been theoretically proposed, for a stationary random noise of an arbitrary distribution being attenuated by passing through the sound insulation barrier. The effect of the statistical properties of input noise emitted from the sound source and the frequency characteristics of the sound insulation barrier on the transmitted noise level or noise energy probability distribution is reflected in each parameter of the cumulative probability distribution expression. The validity of this theoretical evaluation method has been experimentally confirmed by applying it to actual noise data. The experimental results are in good agreement with theoretical curves.

Research on the statistical evaluation of sound insulation barriers is still in an early stage of study. The paper has focused only on some fundamental aspects. There still remain many problems when considering actual cases, and this will be a future study.

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